# The Limits of Thought—and Beyond

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### 1. Introduction: the world beyond thought 1

There was a time when, following Aristotle, most educated people thought that the cosmos (the totality of space and everything in it) was finite and bounded. This changed in the sixteenth and seventeenth centuries under the attack of various radical thinkers. Even in Antiquity, however, the view had its critics: notably Archytas and the later atomists (see Chapter 8 of Sorabji 1989 and Bailey 1970). A persuasive argument used against Aristotle went as follows. The cosmos cannot be bounded. For a boundary has things on both sides of it. Hence, if there were a boundary to the cosmos there would have to be something on the far side of it. But this, by definition, is impossible.<sup>2</sup>

The ultimate fate and validity of this argument I do not intend to discuss. I give it to introduce a similar argument concerning not the universe of space but the universe of thought. Suppose that there were some boundaries to the conceivable, some things that were too complex or recondite to be conceived of (maybe due to the finitude of our brains). Then there must be things on the far side of the boundary, and these must be inconceivable. But to conceive of the boundary is to conceive of things on either side of it; and for the far side this is, by definition, impossible.

We have here an argument for idealism—at least in one sense of that word. A realist—in the corresponding sense—holds that there are things which exist, and which are what they are, quite independently of thought. They would exist even if they were not thought about. It must be possible, therefore, for there to be things that are not conceived of, indeed, inconceivable. But then there must be a boundary between that which is conceivable and that which is not. And this is precisely what this argument rules out.

<sup>&</sup>lt;sup>1</sup> It would seem fair to warn the reader that this paper presupposes dialetheism (the view that contradictions may be true) or, at least, its intelligibility. I make no attempt to defend this here. For a defence see Priest (1987). As well as presupposing dialetheism, the paper can also be seen as supporting it, at least to the extent that it gives arguments for certain contradictions which, I maintain, are difficult to break. I have learned from discussions that though many people do want to break them, there is no consensus about how this should be done. To do justice to the numerous ways that people have suggested would lengthen this paper inordinately, thus completely obscuring its main thrust. I have therefore not attempted this here.

<sup>&</sup>lt;sup>2</sup> As Epicurus put it: "The Universe is boundless. For that which is bounded has an extreme point: and the extreme point is seen against something else. So that as it has no extreme point, it has no limit; and as it has no limit it must be boundless and not bounded" (Bailey 1970).

## 2. Berkeley's "master argument"

This argument is related to Berkeley's famous argument for idealism in the first of the *Dialogues Between Hylas and Philonous*.<sup>3</sup> Berkeley, of course, believed that there is no existence independent of *perception*, rather than *conception*; but for reasons of his own he tends to run these things together. After a number of arguments concerning primary and secondary properties and the notion of matter, Philonous (Berkeley's mouthpiece) says that he is prepared to rest his case on a simple "master argument". He challenges Hylas (his foil) to conceive of something existing without being thought about. When Hylas claims to do just this, Philonous points out that he (Hylas) is thinking about the thing in question. What he has been asked to do is therefore impossible: it is not possible to think of things that are not conceivable, indeed, conceived. Realism is, therefore, quite literally, inconceivable.

The arguments I have been describing belong to a family. This is characterised by the fact that each of its members runs a *reductio* of some kind on the claim that there are things beyond conception or—what seems to be equivalent to this—boundaries to thought. It would be a large task to map all the members of the family, their interconnections, adequacies and exact relevance for various idealisms; and I shall not try here, since my interest at present is not in idealism *per se*, but in a more general notion: the limits of thought.

Let me, however, single out one argument from the family. It can be put simply as follows. (I emphasise that it is *not* Berkeley's argument, which uses not only the notion of conceiving an object but also the notion of conceiving *that* something.)

- (1) Suppose that there is something that is inconceivable.
- (2) Then this thing that is inconceivable is inconceivable.
- (3) But the thing that is inconceivable *is* conceivable (indeed, we have just conceived it in talking about it).
- (4) Thus, our supposition is incorrect: nothing is inconceivable.

I note that the argument works just as well if we replace "conceivable" with "conceive", or even "conceived by me (now)", making for much stronger conclusions.

For those who have a taste for formal logic I formalise the argument as follows, using Cx as "x is conceivable" and an indefinite description operator,  $\varepsilon$  (so that " $\varepsilon x \varphi$ " refers to a selected object that satisfies  $\varphi$ ).

(1)  $(\exists x) \neg Cx$  Supposition (2)  $\neg C\varepsilon x \neg Cx$  From (1) (3)  $C\varepsilon x \neg Cx$  Premise

(4)  $\neg (\exists x) \neg Cx$  From (2) and (3) by reductio, discharging (1)

There are three obvious points of attack on this argument. The first is to challenge the validity of the inference from (1) to (2). This is a valid inference in Hilbert's

<sup>&</sup>lt;sup>3</sup> Pp. 183-4 of Berkeley 1962. Another version of the argument is given by Berkeley in §23 of the *Principles*.

 $\varepsilon$ -calculus and similar accounts of indefinite descriptions.<sup>4</sup> It might be objected that the inference involves quantification into, and descriptions within, the scope of an operator, C, that may be thought to be intensional; and, if so, this might be thought to be problematic. However, I take conceiving, as used in this argument, to be extensional in the relevant sense. Specifically, if someone conceives of x and x = y then they conceive of y—whether or not they are aware of it. Notice that even those who hold that conceiving is mainly (primarily, normally) intensional cannot deny that there is a notion of conceiving that is extensional in this sense; for this can be defined in terms of the intensional sense. Object x is (extensionally) conceived iff for some description (sense, notion, or whatever) x is (intensionally) conceived under that description (sense, notion). This step is therefore correct.

The second point of attack is the premise (3). The justification for this is a simple thought experiment. I ask you to consider an object which is not being conceived. You do so. Indeed, in the relevant sense you can conceive of anything that is referred to by a simple grammatical noun-phrase of English, just because you understand it. By mentally rehearsing the phrase you bring the object it refers to before the mind in the required sense. Of course the thing in question is being conceived of intensionally, under the appropriate description (in this case, "an object that is not being conceived of"). But this does not invalidate the argument since, by the definition of the previous paragraph, intensional conceiving entails extensional conceiving in the required sense. Thus, this premise seems to be correct.

The third, and final step at which one might fault the argument is the reductio inference to (4). I will leave this for the time being and return to it towards the end.<sup>5</sup>

#### 3. Kant's world-series

The boundary between thought and the thought-independent world—if such there be—provides one of the limits of thought. There are others. In the chapter called "The Antinomy of Pure Reason" in The Critique of Pure Reason, Kant (1929) argues that Reason forces us to construct "the unconditioned unity of the objective conditions of the [field of] appearance" (A406, B443); by which he means the limit you get when you take some thought-operation and, starting with some object or event, apply it iteratively as often as possible. As he puts it:

<sup>5</sup> Some may find the argument problematic because of its use of indefinite descriptions. However, these may be traded in for definite descriptions. We merely order the objects (maybe using spatio-temporal coordinates and—if necessary—the Axiom of Choice) and consider the least inconceivable object in this enumeration.

<sup>&</sup>lt;sup>4</sup> Such as that in Priest (1979). As both John Bacon and Vladimir Smirnov pointed out to me, the inference from (3) to (4) is also valid in Hilbert's  $\varepsilon$ -calculus based on classical logic. If this logic were correct there would therefore be a very swift argument to (4). However, the inference fails if the theory of descriptions is based on a suitable paraconsistent logic. To see this, suppose that some things are C and only C, some things are  $\neg C$  and only  $\neg C$ , and some things are both. It follows that  $\forall xCx$  is just plain false. " $\varepsilon x \neg Cx$ " denotes one of the things that satisfies  $\neg C$ , but if it also satisfies C then  $C \in x \neg Cx$  is true (and false too; but at least true). I leave it as an exercise for those readers who wish it to turn this example into a formal counter-model in the semantics of Priest (1987).

Reason does not really generate any concept. The most it can do is *free* a concept of the *understanding* from the unavoidable limitations of possible experience, and so to endeavour to extend it beyond the limits of experience. This is achieved in the following manner. For a given conditioned, reason demands on the side of conditions—to which as the conditions of synthetic unity the understanding subjects all appearances—absolute totality, and in doing so converts the category into a transcendental idea. For only by carrying the empirical synthesis as far as the unconditioned is it enabled to render it absolutely complete; and the unconditioned is never to be met with in experience, but only as an idea. Reason makes this demand in accord with the principle that if the conditioned is given, the entire sum of conditions and consequently the absolutely unconditioned ... is given. (A409, B436; all italics are original)

What Kant has in mind can be seen most clearly with an example. Take any event,  $e_0$ , and apply to it the condition (operator) event temporally prior to. This generates a new event  $e_1$ , prior to  $e_0$ . Now apply the operator to  $e_0$  and  $e_1$  to get a new event,  $e_2$ , prior to  $e_0$  and  $e_1$ ; and so on. The unconditioned (limit) is the result of applying the operator as far as possible. This example is, in fact, from the first of Kant's Antinomies. For reasons of tectonic that need not concern us, Kant supposes that there are exactly four operations (conditions) which generate four limits (unconditioneds). The other three are, respectively: a material part; a cause; a necessary condition. The corresponding limits may be the totality generated (as in the First Antinomy) or the result of applying the operation until it can be no further applied (as in the Second) (A417, B445). But in any case, it is important that the limit should be taken as a completed whole (A411, B438). I note that the operations in question are not really thought-operations, though Kant thought they were because of his Transcendental Idealism.

Now, the significant fact about each of these limits, according to Kant, is that by apparently legitimate reasoning we may establish that it has inconsistent properties. In the four cases the limits are (i) finite and infinite; (ii) simple and composite; (iii) caused and free; (iv) necessary and contingent. (Only the first antinomy is formulated explicitly in this way, but an examination of the arguments shows that they hinge on the inconsistencies in question.) As Kant puts it in the *Prolegomena*:

...there arises an unexpected conflict which never can be removed in the common dogmatic way; because the thesis, as well as the antithesis, can be shown by equally clear, evident, and irresistible proofs... (1950, 52a)

Unfortunately for Kant, most modern readers find his arguments all too resistible. Indeed, the arguments used to establish the contradictions appear either question-begging or fallacious. Had Kant, however, used a *genuine* thought operation, he could have found a limit that *was* contradictory. Consider the operation *thought* of (by which I mean the content, not the act). Now take an object, say Ayer's Rock, and apply the operation to it iteratively to generate the following sequence: Ayer's Rock; the thought of Ayer's Rock; the thought of (Ayer's Rock and the thought of Ayer's Rock); the thought of the three previous things... We know, as Kant did not, that this can be iterated into the transfinite in a natural way. Thus, having generated all the thoughts of finite order, we can apply the operator to give the thought of all thoughts of finite order, and so on. This is a subtlety, however.

The important thing is that the operation is iterated as far as possible, however far that is. The result is, by definition, such that it can be no further applied. But now consider the limit, that is, the totality of all thoughts generated in this way. By definition, it is such that it is impossible to apply the operator to it (or the operator would not have been applied as far as possible); hence there is no thought of it. Yet it clearly is possible to think of this totality: we have just done so. Contradiction: the totality is the limit of thought, but also transcendable. I will refer to this example as the Fifth Antinomy.

The fact that Kant may have been wrong about the details should not, therefore, be allowed to detract from his central profound insight: that antinomies may arise at the limit of a thought construction.

### 4. The Hegelian turn

Kant's analysis of the problem of the Antinomies was less impressive than his formulation of it. Most notable amongst its failings is that its details are notoriously obscure. But the root of the antinomies is, Kant assures us, the confusing of phenomena with noumena, and so the taking one or the other to behave incorrectly (A498-9, B526-7). I do not want to go into details here. I note only that when Kant attempts to work out the details he gives two conflicting solutions to the Antinomies. The first (A498, B526, ff) is meant to apply to all the Antinomies, and is to the effect that the limit does not exist (to suppose so is to take phenomena for noumena). The second (A533, B561, ff) is specific to the Third and Fourth Antinomies, and is to the effect that the limit exists, but the pertinent categories do not apply to it (to suppose so is to take noumena for phenomena).

The crucial question now becomes whether Kant can sustain the categorical distinction between phenomena and noumena, and, in particular, the claim that the Categories apply only to phenomena. I think he cannot; indeed, I think that this thesis cannot even be stated without violating it. But this is too long a story to go into here. I raise the point only because it is that onto which Kant's successors latched. Hegel, in particular, argued that it was not possible to sustain the thesis that there is an essential difference between phenomena and noumena. (In particular, things in themselves, an important kind of noumena, are just as knowable as phenomena.) It follows that Kant's solutions to the Antinomies collapse. Hegel drew the appropriate conclusion: the limits of thought involved in the Antinomies are dialetheic objects: objects which truly have inconsistent properties:<sup>7</sup>

> True, Kant's expositions of the antinomies of pure reason, when closely examined ... do not deserve any great praise; but the general idea on

<sup>&</sup>lt;sup>6</sup> "The Thing-in-itself ... expresses the object when we leave out of sight all that consciousness makes of it, all its emotional aspects, and all specific thought of it. It is easy to see what is left—utter abstraction.... Hence one can only read with surprise the perpetual remark that we do not know the Thing-in-itself. On the contrary there is nothing we can know so easily." (Hegel 1975, §44)

<sup>&</sup>lt;sup>7</sup> Page references to, and quotations from, the *Logic* are taken from Miller (1969). All italics are original.

which he based his exposition and which he vindicated is ... the necessity of the contradiction which belongs to the nature of thought determinations: primarily, it is true, with the significance that these determinations are applied by reason to things in themselves, but their nature is precisely what they are in reason ... (Logic, p. 56)

Hegel takes up the nature of these contradictions again when he discusses (quantitative) infinity in the *Logic*. He notes that in the Antinomies:

[t]he thesis and the antithesis and their proofs ... represent nothing but the opposite assertions, that a *limit is*, and that the limit equally is a *sublated* one; that the limit has a beyond, with which it stands in a relation, and beyond which it must pass ... (*Logic*, p. 237)

Recall the Fifth Antinomy, and you will see that this is exactly right. The argument shows that there is a limit (generated by a certain operation), which is also not a limit (since the operation can be applied again), which is exactly what Hegel means.

According to Hegel there are two sorts of infinities, true and false. The false (or "spurious") infinity is the unboundedness of an infinite progression. But such a progression is only:

the *problem* of attaining the infinite, not the actual reaching of it; it is the perpetual generation of it, but it does not get beyond quantum, nor does the infinite become positively present. (*Logic*, p. 277)

By contrast, a true infinity is a totality which is both bounded and transcends that bound, the dialectical unity of limit and its negation. As he puts it:

[t]he self-sublating of this [false] infinite and of the finite as a single process—this is the true or genuine infinite. (Logic, p. 139)

The limit objects are, as we see, true infinities.

# 5. Absolute infinity

Historically speaking, the Kant/Hegel insight, that certain limit objects are inconsistent, was lost for the best part of 100 years. Commentators on Kant concentrated on the details of particular antinomies, rather than their underlying problematic. And few took Hegel's dialectic seriously—or, at least, if they did, they were more interested in applying it to other things. The phenomenon returned with a vengeance, however, in an unexpected place—set theory. (Though, with hindsight, this appearance may be obvious, since set theory is, amongst other things, the mathematical theory of the limits of iterated operations.) This produced contradictions of just the kind that fit the Kantian problematic.<sup>8</sup>

Consider, for example, Burali-Forti's paradox. Take the operation *least ordinal greater than*. Starting with zero iterate the operation as far as is possible (and so into the transfinite). Now consider the limit of this procedure, the collection of all ordinals. By construction, it is impossible to apply the operator again to gen-

<sup>&</sup>lt;sup>8</sup> The connection between Kant's Antinomies and the set-theoretic paradoxes has been noted by a number or people including Hessenberg, Zermelo, Fraenkel and Martin. For references and discussion, see Chapter 6.2 of Hallett (1984).

<sup>&</sup>lt;sup>9</sup> For a fuller account of the following paradoxes see, e.g. Fraenkel (1973).

erate a new ordinal. But it is not difficult to show that the collection itself is (or has, in some constructions) an ordinal, which is the least ordinal greater than all ordinals. Hence, the operator can be applied again. Thus this limit is inconsistent.

Alternatively, consider the operation of taking the power-set (set of all subsets), which produces a set of greater (cardinal) size. Starting with the empty set, apply this operation as often as possible (and so into the transfinite). Now consider the limit of this construction. (Set-theorists call this the Cumulative Hierarchy.) It is impossible to apply the power set operator to this to obtain a bigger set since we have applied it as often as possible. Yet taking the power-set is an iterable operation, and hence we can apply it to get a bigger set. This is a version of Cantor's paradox (of which Russell's is a variant).

As early as 1905, Russell had found the general scheme to which these paradoxes belong:

> Given a property  $\phi$  and a function f, such that, if  $\phi$  belongs to all members of u, f(u) always exists, has the property  $\phi$ , and is not a member of u; then the supposition that there is a class w of all terms having the property  $\phi$  and that f(w) exists leads to the contradiction that f(w) both has and has not the property  $\phi$ . (Russell 1973, p. 142)

In other words, if applying f to a collection of  $\phi$ -things always gives a new  $\phi$ thing, then applying it to the collection of all  $\phi$ -things gives rise to a contradiction. In the Burali-Forti paradox  $\phi$  is the property "is an ordinal", and the generating function, f, is the function the least ordinal greater than. In Cantor's paradox,  $\phi$  is the property "is a well founded set" and f is the function the power set of. And in the Fifth Antinomy,  $\phi$  is the property 'is a "well founded" thought of Ayer's Rock' and f is the function the thought of.  $^{10}$ 

As is implicit in Russell's observation, there are essentially only two ways of avoiding the contradiction. The first is to say that  $\{x; \phi x\}$  does not exist; the second is to say that it exists, but f cannot apply to it. These are exactly Kant's two solutions, of course.

Modern set theorists are inclined to have it both ways, by distinguishing between sets and proper classes. In paradoxical cases the set  $\{x; \phi x\}$  does not exist; the proper class  $\{x; \phi x\}$  exists but cannot be a member of anything, and, a fortiori, cannot be the object of more complex operations (again, for details, see Fraenkel

<sup>10</sup> This makes the isomorphism between the Kantian and the set-theoretic Antinomies crystal clear. Martin clearly has the same point in mind when he says of a typical paradox of absolute infinity:

> ...we are concerned with the concept of totality. I consider all classes and form the concept of their totality, namely the class of all classes. But then I regard this totality as an element in the formation of a new class. A concept of a totality is again formed and this new totality is again used as an element. This conflict between concluding and beginning anew, between forming a totality and using this totality as a new element, is the actual ground of the antimony. It is this conflict that gives the connection with the Kantian antinomies. Kant saw quite clearly that the antinomies rest on this antithesis between making a conclusion and going beyond the conclusion. In principle, this had already been seen by Archytas, when he wanted to go to the end of the world and stretch out his arm.... (1955, p. 55)

Bennett (1974, p. 115), criticising Martin, says that there is "no significant similarity" between the Antinomies and the set-theoretic paradoxes. This would appear to be because he has lost the wood for the trees (see, e.g. p. 285).

1973). This distinction is quite unsatisfactory, however. The criterion for being a proper class is quite clear: size; but there exists *no* independent argument as to why proper classes cannot be members of other classes. Indeed, if they are genuine totalities, there is *no* reason why we cannot consider, e.g., their singletons. Indeed, proper classes will not bear the theoretical load that is placed on them; for parts of mathematics (e.g. category theory, semantics) require us to do just that (for details, see Priest 1987, Chapter 2, especially §§3 and 4).

It should also be noted that neither kind of solution has much plausibility in the case of the Fifth Antinomy. To claim that the limit is a proper class does not seem to help much. For the fact that something is a proper class surely does not stop us thinking about it. And even to claim that the limit does not exist doesn't seem to help much; for there is obviously *some* sense in which one can think of non-existent objects, such as Pegasus. Thus, we see that the totalities and operations involved appear to be inherent in thought. And nothing save prevarication can prevent us applying the operations to the totalities to generate contradiction.

## 6. Referring and conceiving

The limit objects which generate the set-theoretic contradictions we have just been looking at were called "absolute infinities" by Cantor; but the paradoxes of Absolute Infinity are not the only paradoxes of set theory. In particular, there are paradoxes of definability. For example, it is not difficult to show that there are more ordinal numbers than are referred to by noun phrases of English. Consider a number which is not referred to (or the least such number, to be more definite). By construction, it is not referred to; but we have just referred to it. Let us say that an object is referable iff there is a noun-phrase that refers to it. Then for reasons that will become clear in a moment, let me spell out the business-end of the argument:

- (1') There is something that is irreferable.
- (2') Then this thing that is irreferable is irreferable.
- (3') But the thing that is irreferable is referable (indeed, we have just referred to it in talking about it).

The paradox is essentially that of Koenig (1905; see the introduction to this in van Heijenoort 1971), and shows that another kind of limit is dialetheic: the limit of what is referred to.

Now let us return to the argument for idealism of §2. This concerns not referability but conceivability. These notions may not be exactly the same, but they would seem to be pretty well coextensional. For if there is a noun-phrase that refers to something then that thing can be conceived (we can use the phrase to single out the target of the thought, as it were); and if something can be conceived, there must be some way of singling it out, and hence some way of referring to it

<sup>&</sup>lt;sup>11</sup> Note that this paradox does not quite fit into Russell's scheme. We may take  $\emptyset$  to be the property "is a definable ordinal", and f to be the function the least number greater than every member of; but if u is a set of definable ordinals, there is no guarantee that f(u) is definable (since u might not be).

(at least by a mental token if not a physical one). Thus, the three lines of Koenig's paradox are essentially the first three lines of the argument for idealism, except that the first line is no longer a supposition, but has an independent proof. This puts the argument in a completely new light. As we have seen, the limit of thought in question here is dialetheic. Thus, lines (2) and (3) of the argument state its properties quite truly. Moreover, line (4) of the argument, the reductio step, can now be seen not to work. For a start, line (1) is no longer a mere supposition: there is an independent argument for it. But anyway reductio is not going to work in this context. If the contradiction involved is true it cannot be used to reduce a subimplicant to absurdity.12

### 7. Conclusion: the limits of thought

I have now finally dealt with the argument for idealism of §2; and as we have seen, it fails. It fails since it falls foul of the fact that it deals with a particular limit of thought, which is a dialetheic object—one which truly has inconsistent properties.

But the preceding discussion illustrates a much more general thesis, the proposal of which is the main point of this paper. The thesis is that there are certain limits to thought whose very notion is dialetheic. Thought, as it were, thinks of these limits, and, in doing so, trips over them; it cannot help but do the impossible and go beyond them (too).

As Hegel, that most insightful but frustratingly obscure commentator, put it in one of his more lucid moments:

> ... great stress is laid on the limitations of thought, reason, and so on, and it is asserted that the limitation cannot be transcended. To make such an assertion is to be unaware that the very fact that something is determined as a limitation implies that the limitation is already transcended. (Logic, p. 134)

Thought can, indeed must, therefore, think beyond its own limits.<sup>13</sup>

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<sup>12</sup> How, exactly, one cashes out this insight depends on a variety of factors, such as the exact details of the underlying logic. (1)&(3) implies a contradiction. If contraposition fails the argument stops there; but given contraposition and the law of excluded middle, we can infer  $\neg$  ((1)&(3)). The argument from this and (3) to  $\neg$  (1) is an instance of the disjunctive syllogism, however, and fails in dialetheic contexts.

<sup>13</sup> This paper had its origins in a series of seminars on the limits of thought that I gave with Uwe Petersen at the University of Western Australia. Though he will certainly not agree with the dialetheic conclusions of this paper, his contribution to its content is substantial. I am also indebted to one of the students in the class, Neil Levi, for his discussion of Hegel on the infinite, and to the editor of Mind for his helpful comments. An earlier version of the paper was given as the Presidential Address at the 1989 meeting of the Australasian Association of Philosophy in Canberra.

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